Calcolo di F^t tramite Jordan

4(i). $J_{\lambda_i,j} = \begin{bmatrix} \lambda_i & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & \lambda_i \end{bmatrix} \in \mathbb{R}^{r_{ij}\times r_{ij}} \overset{\lambda_i \neq 0}{\implies} J^t_{\lambda_i,j} = (\lambda_i I + N)^t, \quad N = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$

note

G. Baggio | Lez. 7: Modi, risposta libera e forzata (t.d.) | 11 Marzo 2021

$$J_{\lambda_{i,j}} = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix} = \lambda_i I + \overbrace{\begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{bmatrix}}^{N}$$

<u>Proposizione</u>: $A, B \in \mathbb{R}^{n\times n}$, $AB = BA$, allora

$$(A+B)^t = \sum_{k=0}^{t} \binom{t}{k} A^{t-k} B^k$$

$$J_{\lambda_{i,j}}^t = (\lambda_i I + N)^t = \sum_{k=0}^{t} \binom{t}{k} (\lambda_i I)^{t-k} N^k \qquad (t > r_{ij}-1)$$

$\lambda_i I, N$ commutano

$$= \binom{t}{0} \lambda_i^t \underbrace{N^0}_{= I} + \binom{t}{1} \lambda_i^{t-1} N + \binom{t}{2} \lambda_i^{t-2} N^2 + \cdots + \binom{t}{r_{ij}-1} \lambda_i^{t-r_{ij}-1} N^{r_{ij}-1}$$

$$N = \begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{bmatrix} \qquad N^2 = \begin{bmatrix} 0 & 0 & 1 & & 0 \\ & \ddots & \ddots & \ddots & 0 \\ & & \ddots & \ddots & 0 \\ & & & & 0 \end{bmatrix} \quad \cdots \quad N^{r_{ij}-1} = \begin{bmatrix} 0 & & & 0 & 1 \\ & \ddots & & & 0 \\ & & \ddots & & \\ & & & & 0 \end{bmatrix}$$

Evoluzione forzata

$$x(t+1) = Fx(t) + Gu(t), \qquad x(0) = x_0$$
$$y(t) = Hx(t) + Ju(t)$$

$$x(t) = x_\ell(t) + x_f(t), \qquad x_\ell(t) = F^t x_0, \qquad x_f(t)\ ??$$
$$y(t) = y_\ell(t) + y_f(t), \qquad y_\ell(t) = HF^t x_0, \qquad y_f(t)\ ??$$

note

$$\begin{cases} x(t+1) = Fx(t) + Gu(t) \\ y(t) = Hx(t) + Ju(t) \end{cases}$$

$$x(1) = Fx(0) + Gu(0)$$

$$x(2) = Fx(1) + Gu(1) = F(Fx(0) + Gu(0)) + Gu(1) = F^2x(0) + FGu(0) + Gu(1)$$

$$x(3) = Fx(2) + Gu(2) = F(F^2x(0) + FGu(0) + Gu(1)) + Gu(2) = F^3x(0) + F^2Gu(0) + FGu(1) + Gu(2)$$

$$\vdots$$

$$x(t) = F^t x(0) + F^{t-1}Gu(0) + F^{t-2}Gu(1) + \cdots + Gu(t-1)$$
$$= \underbrace{F^t x(0)}_{x_\ell(t)} + \underbrace{\sum_{k=0}^{t-1} F^{t-1-k} G u(k)}_{x_f(t)} =$$
$$= F^t x(0) + \underbrace{\begin{bmatrix} G & FG & F^2G & \cdots & F^{t-1}G \end{bmatrix}}_{R_t} \begin{bmatrix} u(t-1) \\ u(t-2) \\ \vdots \\ u(0) \end{bmatrix}$$

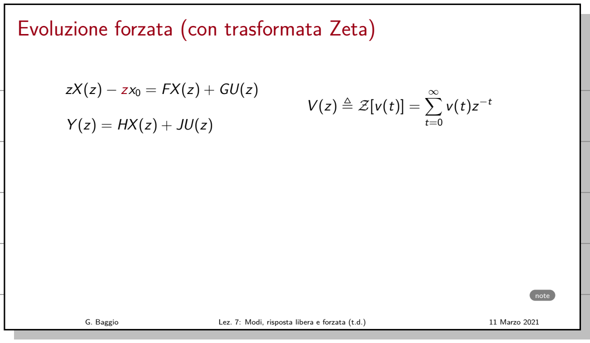
matrice di raggiungibilità in t passi

$$y(t) = Hx(t) + Ju(t) = \underbrace{HF^t x(0)}_{y_\ell(t)} + \underbrace{\sum_{k=0}^{t-1} HF^{t-1-k} G u(k) + Ju(t)}_{y_f(t)}$$

$$w(t) = \begin{cases} HF^{t}G & t \geq 1 \\ J & t = 0 \end{cases}$$ risposta impulsiva

$$y_f(t) = [w \underset{\uparrow}{*} u](t)$$

convoluzione discreta



$$\begin{cases} x(t+1) = F x(t) + G u(t) \\ y(t) = H x(t) + J u(t) \end{cases}$$

Trasformata Zeta:

$$V(z) = \mathcal{Z}[v(t)] = \sum_{t=0}^{\infty} v(t) z^{-t}$$

$$\mathcal{Z}[v(t+1)] = z V(z) - z v(0)$$

$$\begin{cases} x(t+1) = F x(t) + G u(t) \\ y(t) = H x(t) + J u(t) \end{cases} \xrightarrow{\mathcal{Z}} \begin{cases} z X(z) - z x(0) = F X(z) + G U(z) \\ Y(z) = H X(z) + J U(z) \end{cases}$$

$$\begin{cases} X(z) = z (zI - F)^{-1} x(0) + (zI - F)^{-1} G U(z) \\ Y(z) = z H (zI - F)^{-1} x(0) + [H (zI - F)^{-1} G + J] U(z) \end{cases}$$

$(x(0) = 0)$

1) $W(z) = \dfrac{Y(z)}{U(z)} = H (zI - F)^{-1} G + J$ Matrice di trasferimento

2) $\mathcal{Z}[x_\ell(t)] = \mathcal{Z}[F^t x(0)] = \mathcal{Z}[F^t] x(0) = z (zI - F)^{-1} x(0)$

$$\boxed{z (zI - F)^{-1} = \mathcal{Z}[F^t]}$$