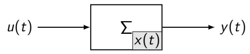
Sistemi lineari tempo-invarianti (LTI) in spazio di stato



Σ lineare e tempo invariante $\qquad x(t) \in \mathbb{R}^n,\ u(t) \in \mathbb{R}^m,\ y(t) \in \mathbb{R}^p$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \in \mathbb{R}^n \qquad u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix} \in \mathbb{R}^m$$

$$y(t) = \begin{bmatrix} y_1(t) \\ \vdots \\ y_p(t) \end{bmatrix} \in \mathbb{R}^p$$

$$\begin{cases} \dot{x}_1 = f_{11} x_1 + \cdots + f_{1n} x_n + g_{11} u_1 + \cdots + g_{1m} u_m \\ \vdots \\ \dot{x}_n = f_{n1} x_1 + \cdots + f_{nn} x_n + g_{n1} u_1 + \cdots + g_{nm} u_m \end{cases}$$

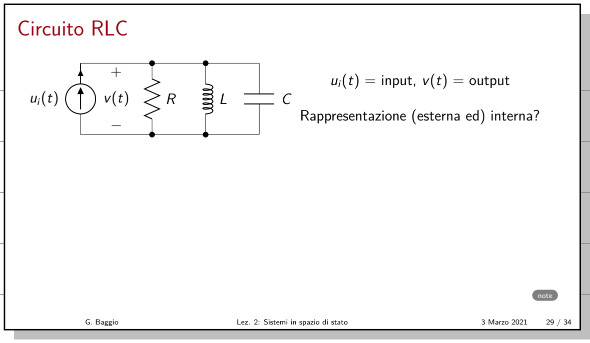
$$F = \begin{bmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \\ f_{n1} & \cdots & f_{nn} \end{bmatrix} \in \mathbb{R}^{n \times n} \qquad G = \begin{bmatrix} g_{11} & \cdots & g_{1m} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nm} \end{bmatrix} \in \mathbb{R}^{n \times m}$$

$\dot{x} = Fx + Gu \qquad F =$ matrice di stato, $G =$ matrice di ingresso

$$\begin{cases} y_1 = h_{11} x_1 + \cdots + h_{1n} x_n + j_{11} u_1 + \cdots + j_{1m} u_m \\ \vdots \\ y_p = h_{p1} x_1 + \cdots + h_{pn} x_n + j_{p1} u_1 + \cdots + j_{pm} u_m \end{cases}$$

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & & \\ h_{p1} & \cdots & h_{pn} \end{bmatrix} \in \mathbb{R}^{p \times n} \qquad J = \begin{bmatrix} j_{11} & \cdots & j_{1m} \\ \vdots & & \vdots \\ j_{p1} & \cdots & j_{pm} \end{bmatrix} \in \mathbb{R}^{p \times m}$$

$y = Hx + Ju \qquad H =$ matrice di uscita, $J =$ matrice di feed-forward



$u_i(t)$ = corrente erogata dal generatore = input

$v(t)$ = tensione ai capi di R, L, C = output

Leggi delle componenti R, L, C:

R) $v_R = R\, i_R$

L) $v_L = L \dfrac{di_L}{dt}$

C) $i_C = C \dfrac{dv_C}{dt}$

Leggi del circuito:

1) $v = v_R = v_L = v_C$

2) $u_i = i_R + i_L + i_C$

2) $$\frac{di_R}{dt} + \frac{di_L}{dt} + \frac{di_C}{dt} = \frac{du_i}{dt}$$

$$\frac{1}{R}\frac{dv_R}{dt} + \frac{v_L}{L} + C\frac{d^2 v_C}{dt^2} = \frac{du_i}{dt}$$

$$\frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v - \frac{1}{C}\frac{du_i}{dt} = 0$$

dominio Laplace

$$s^2 V(s) + \frac{s}{RC}V(s) + \frac{1}{LC}V(s) = \frac{s}{C}U_i(s)$$

$$G(s) = \frac{V(s)}{U_i(s)} = \frac{s/C}{s^2 + s/RC + 1/LC}$$

Rappresentazione interna:

variabili di stato = $\begin{cases} \text{tensioni su } C \\ \text{correnti su } L \end{cases}$

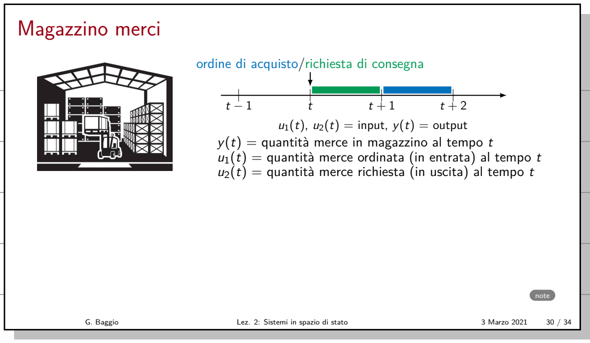
$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad x_1(t) = v_C(t) \qquad x_2(t) = i_L(t)$$

$$\dot{x}_1 = \frac{dv_C}{dt} = \frac{1}{C} i_C = \frac{1}{C}\left(u_i - i_R - i_L\right)$$

$$= \frac{1}{C}\left(u_i - \frac{\overset{\overset{v_C}{\shortparallel}}{v_R}}{R} - x_2\right) = \frac{1}{C}\left(u_i - \frac{x_1}{R} - x_2\right)$$

$$\dot{x}_2 = \frac{di_L}{dt} = \frac{1}{L} v_L = \frac{1}{L} v_C = \frac{1}{L} x_1$$

$$\begin{cases} \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ 1/L & 0 \end{bmatrix}}^{F} x + \overbrace{\begin{bmatrix} 1/C \\ 0 \end{bmatrix}}^{G} u \\ y = v = x_1 = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{H} x + \underbrace{0}_{J} \cdot u \end{cases}$$



$y(t) =$ quantità di merce al tempo $t =$ output

$\left.\begin{array}{l} u_1(t) = \text{quantità di merce ordinata al tempo } t \\ u_2(t) = \text{quantità di merce richiesta al tempo } t \end{array}\right\}$ input

Rappresentazione esterna:

$$y(t+1) = y(t) + u_1(t-1) - u_2(t)$$

$$y(t+1) - y(t) - u_1(t-1) + u_2(t) = 0$$

trasformate Zeta $\longrightarrow$

$$z\,Y(z) - Y(z) = z^{-1}\,U_1(z) - U_2(z)$$

$$G(z) = \begin{bmatrix} \frac{Y(z)}{U_1(z)} & \frac{Y(z)}{U_2(z)} \end{bmatrix} = \begin{bmatrix} \frac{z^{-1}}{z-1} & \frac{-1}{z-1} \end{bmatrix} \qquad \text{(F. d. T.)}$$

Rappresentazione interna:

$$x_1(t) = y(t), \quad x_2(t) = u_1(t-1) \qquad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$x_1(t+1) = y(t+1) = y(t) + u_1(t-1) - u_2(t) = x_1(t) + x_2(t) - u_2(t)$$

$$x_2(t+1) = u_1(t)$$

$$\begin{cases} x(t+1) = \overbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}^{F} x(t) + \overbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}^{G} u(t) \\ y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{H} x(t) + \underbrace{0}_{J} \cdot u(t) \end{cases}$$